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A New Method for C^k-Surface Approximation From a Set of Curves, With Application to Ship Track Data in the Marianas Trench[1]

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We introduce a surface approximation technique to address the problem of fitting a surface to a given set of curves. The originality of the method lies in its ability to take into account the continuous aspect of the data, and also in the possibility to arbitrarily select the regularity (C^0, C^1, or higher) of the approximant obtained. We demonstrate the efficiency of the approach by constructing a bathymetry map of the Marianas trench based upon a set of SONAR (SOnic Navigation And Ranging) bathymetry ship track data.

KEY WORDS: surface fitting, bathymetry, splines, finite elements.

INTRODUCTION

The problem of constructing a smooth surface from a given set of curves appears in many instances in geophysics and geology. One can think for instance of the problem of reconstructing seafloor surfaces from SONAR ship track bathymetry data, as is studied in this paper. Another example is the construction of a Digital Elevation Model from a given set of topography isolines (isolevels). Classical algorithms used to solve this class of problems usually select points on the curves to define a Lagrange data set, and subsequently make use of classical spline functions (e.g., de Boor, 1978; Laurent, 1972; Schumaker, 1981), bivariate splines (Lai and Schumaker, 1998, 1999; von Golitschek and Schumaker, 1990), or spline functions

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in Hilbert spaces (e.g., Arcangéli, 1986; Duchon, 1977). In the available literature, to our knowledge there are no classical methods that explicitly take into account the continuous aspect of the curves that constitute the data set.

In this paper, we propose an approximation method that honors the continuous aspect of the data. We use a fidelity criterion to the data, of integral type, which is based upon a L^2-norm (e.g., Apprato and Arcangéli, 1991). In this respect, the method is related to the surface approximation technique introduced in the context of partial data sets by Apprato, Gout, and Sénéchal (2000) and Apprato and others (2000).

DEFINITION OF THE PROBLEM

The problem of surface approximation from a given set of curves can be posed as follows: from a finite set of open subsets F_j, $j = 1, \ldots, N$ (the bathymetry ship track curves in our case) in the closure of a bounded nonempty open set $\Omega \subset \mathbb{R}^2$, and from a function f defined on $F = \cup_{j=1}^{N} F_j$, construct a regular function Φ on Ω approximating f on F, i.e.:

$$\Phi_{|F} \simeq f_{|F}. \tag{1}$$

We can assume that Ω is a connected set, with a Lipschitz-continuous boundary (following the definition of Necas, 1967), that for any integer j, with $j = 1, \ldots, N$, F_j is a nonempty connected subset in F, and that, for simplicity, f is the restriction on F of a function, still denoted by f, that belongs to the usual Sobolev space $H^m(\Omega)$, with $m \geq 2$. We also assume that the approximant Φ belongs to $H^m(\Omega) \cap C^k(\bar{\Omega})$, with $k = 1$ or 2, where $\bar{\Omega}$ is the closure of Ω. The main interest of such a regularity for Φ is that it allows one to obtain a final surface that can later be used directly as an input model in a different application, such as ray tracing, image synthesis, or numerical simulation (e.g., Komatitsch and Tromp, 1999; Komatitsch and Vilotte, 1998) for instance.

When $m > k + 1$, the interpolation problem $\Phi_{|F} = f_{|F}$ has an infinity of solutions because of the continuous embedding of $H^m(\Omega)$ in $C^k(\bar{\Omega})$. After discretization of the data set, we can obtain a solution using for instance the spline approximation developed by Duchon (1977). Unfortunately, Duchon's theory leads to linear systems whose order increases rapidly with the number of data points, which makes the method inefficient in the case of large data sets. Franke (1982) proposed to use overlapping segments to overcome this problem. We can also obtain a solution using an interpolation method. Let us define, for any $v \in H^m(\Omega)$, $\rho v = v_{|F}$, and let us introduce the convex set $K = \{v \in H^m(\Omega), \rho v = \rho f\}$. Then we consider the minimization problem of finding $\sigma \in K$ such that for any $v \in K$,

$$|\sigma|_{m,\Omega} \leq |v|_{m,\Omega}, \tag{2}$$

where

$$|v|_{m,\Omega} = \left(\sum_{|\alpha|=m} \int_\Omega (\partial^\alpha v)^2 \, dx \right)^{1/2}, \tag{3}$$

with $\alpha = (\alpha_1, \alpha_2) \in \mathbb{N}^2$, $|\alpha| = \alpha_1 + \alpha_2$, $x = (x_1, x_2)$, and $\partial^\alpha v = \frac{\partial^{|\alpha|} v}{\partial x_1^{\alpha_1} \partial x_2^{\alpha_2}}$. If $L^2(F)$ is equipped with the usual norm

$$\|v\|_{0,F} = \left(\sum_{j=1}^{N} \int_{F_j} v^2(x) \, dx \right)^{1/2}, \tag{4}$$

and under the hypothesis that for any $p \in P_{m-1}(\overline{F})$, $p_{|F} = 0 \;\Rightarrow\; p \equiv 0$, we know, based upon a compactness argument (Necas, 1967), that the function $|||\cdot|||$ defined by

$$|||u||| = \left(\|\rho u\|_{0,F}^2 + |u|_{m,\Omega}^2 \right)^{1/2} \tag{5}$$

is a norm on $H^m(\Omega)$, which is equivalent to the usual norm

$$\|u\|_{m,\Omega} = \left(\sum_{|\alpha| \leq m} \int_\Omega (\partial^\alpha v)^2 \, dx \right)^{1/2}. \tag{6}$$

Then the solution σ of the interpolation problem (2) is the unique element of minimal norm $|||\cdot|||$ in K that is convex, nonempty, and closed in $H^m(\Omega)$. Hence we could take the solution $\Phi = \sigma$ when $m > k + 1$. Unfortunately, it is often impossible to compute σ using a discretization of problem (2), because in a finite dimensional space, it is generally not possible to satisfy an infinity of interpolation conditions. Therefore, to take into account the continuous aspect of the data $f_{|F}$, we instead choose to define the approximant Φ as a fitting surface on the set:

$$\{(x_1, x_2, x_3) \in \mathbb{R}^3, x_3 = f(x_1, x_2), (x_1, x_2) \in F_j, \; j = 1, \ldots, N\}. \tag{7}$$

The use of spline functions is common in surface approximation (e.g., Mitasova and Mitas, 1993; Wahba, 1990; Wessel and Bercovici, 1998). In this paper, we propose to construct a "smoothing D^m-spline," as defined by Arcangeli (1986), that will be discretized in a suitable piecewise-polynomial space. The use of such spline functions has been shown to be efficient in the context of geophysical applications such as Ground Penetrating Radar data analysis (Apprato, Gout, and Sénéchal, 2000) or the creation of Digital Elevation Models describing topography (Gout and Komatitsch, 2000). Comparisons between spline functions and classical kriging can be found in Dubrule (1984).

DESCRIPTION OF THE METHOD

Let us in this section present the theoretical aspects of the method. We first introduce a functional J_ε, that we shall minimize, defined on $H^m(\Omega)$ by

$$J_\varepsilon(v) = \|v - f\|_{0,F}^2 + \varepsilon |v|_{m,F}^2, \tag{8}$$

where $\varepsilon |v|_{m,F}^2$ is a smoothing term, $\varepsilon > 0$ being a classical smoothing parameter. The key idea here is that the fidelity criterion to the data $\|v - f\|_{0,F}^2$ honors their continuous aspect. We now need to numerically estimate this L^2-norm, which is done using a quadrature formula. In this regard, the approach is quite different from more classical techniques that usually simply make use of a large number of data points on F in order to solve the approximation problem.

For any integer j, $j = 1, \ldots, N$, and any $\eta > 0$, let $\{\zeta_i\}_{1 \le i \le L}$ be a set of $L = L(j)$ distinct points $\zeta_i = \zeta_i(j)$ of $\overline{F}_j$ such that

$$\max_{1 \le i \le L-1} \delta(\zeta_i, \zeta_{i+1}) \le \eta, \tag{9}$$

where δ is the Euclidean distance in $\mathbb{R}^2$. This relation implies that the distance between two consecutive ζ_i is bounded by η; it also allows one to study the convergence of the approximation when $\eta \to 0$. The ζ_i will also be the nodes of a numerical integration formula. Let us also introduce a set $\{\lambda_i\}_{1 \le i \le L}$ of real numbers (that will be the weights of a quadrature formula) such that $\lambda_i = \lambda_i(j) > 0$, and let us define, for any $v \in C^0(\overline{F}_j)$, $\forall \eta > 0$,

$$\ell_j^\eta(v) = \sum_{i=1}^{L} \lambda_i v(\zeta_i), \tag{10}$$

and for any $v \in C^0(\overline{F})$

$$\ell(v) = \sum_{j=1}^{N} \ell_j^\eta(v). \tag{11}$$

In all that follows, we will suppose that, for any $v \in H^m(\overline{F})$, any $\eta > 0$, there exists $C > 0$ such that

$$\left| \ell_j^\eta(v^2) - \|v\|_{0,F_j}^2 \right| \le C\eta \|v\|_{m,\Omega}^2. \tag{12}$$

When this hypothesis is satisfied, one can consider ℓ as a theoretical quadrature formula for $\|\cdot\|_{0,F}^2$. Note that if we have only one curve (i.e., $N = 1$ above) and F is represented by a unique equation $x_2 = a(x_1) : x_1 \in \Delta$, and if we define the norm $\|\cdot\|_{0,F}$ by

$$\|v\|_{0,F}^2 = \int_\Delta v^2(x_1, a(x_1))(1 + a'^2(x_1 0))^{1/2}\, dx_1, \tag{13}$$

then we can also define

$$\ell(v) = \frac{1}{2}\delta(\xi_1, \xi_2)v(\xi_1) + \frac{1}{2}\sum_{i=2}^{L-1}[\delta(\xi_{i-1}, \xi_i) + \delta(\xi_i, \xi_{i+1})]v(\xi_{i1})$$
$$+ \frac{1}{2}\delta(\xi_{L-1}, \xi_L)v(\xi_L), \tag{14}$$

and verify that $\ell(v)$ is a quadrature formula for the curvilinear integral $\int_F v(x)\, ds$. Note also that in most applications the F_j are polygonal curves, and one can, therefore, use a classical quadrature formula (e.g., Gout and Guessab, 2001).

Let $\tilde{\Omega}$ be a bounded polygonal open set in $\mathbb{R}^2$ such that $\Omega \subset \tilde{\Omega}$, and let us define a typical size h and a mesh size h_K such that for any $h > 0$, $\tilde{\mathcal{Q}}_h$ is a quadrangulation on $\tilde{\Omega}$ constructed using elements K whose size h_K is smaller than h (Fig. 1). Let us also consider the subset Ω_h that is the interior of the union of the rectangles K of $\tilde{\mathcal{Q}}_h$ such that $K \cap \Omega \neq \emptyset$ (i.e., the union without its exterior

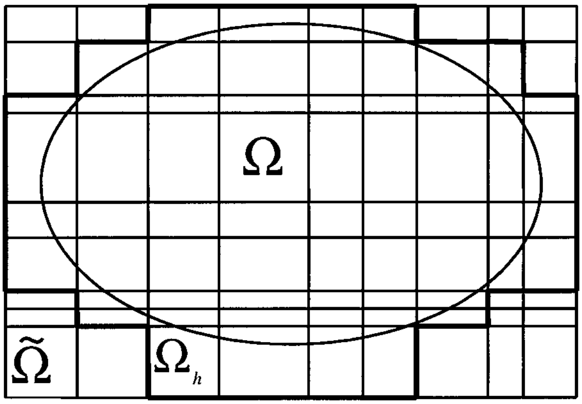

Figure 1. Definition of the sets Ω, Ω_h, and $\tilde{\Omega}$ used in our numerical algorithm. Ω is the open set on which we wish to define the approximant, $\tilde{\Omega}$ is a polygonal open set containing Ω, and Ω_h is a set of quadrangles contained in $\tilde{\Omega}$ and containing Ω. Note that there is no good automatic way of choosing the typical size of the finite elements in the grid. The selection must be done manually based upon the characteristics of the data set under study.

edge). We then introduce the functional $\tilde{J}_{\varepsilon,h}$ defined on $H^m(\Omega_h)$ by

$$\tilde{J}_{\varepsilon,h}(v_h) = \ell[(v_h - f)^2] + \varepsilon |v_h|^2_{m,\Omega_h}, \tag{15}$$

and consider the minimization problem of finding $\Phi \in H^m(\Omega_h)$ such that

$$\tilde{J}_{\varepsilon,h}(\Phi) = \min_{v_h \in H^m(\Omega_h)} \tilde{J}_{\varepsilon,h}(v_h). \tag{16}$$

In order to compute a discrete approximant Φ, we could use any finite dimensional space, but for practical reasons we choose a polynomial space. We could use a Bézier-polynomial expansion, but instead we select a finite element representation of Φ similar to that used in Apprato, Gout, and Sénéchal (2000), in order to be able to choose the regularity that we want (C^0, C^1, or higher) for the solution. The use of finite elements also allows us to obtain a very small sparse linear system and makes the study of the approximation error easier.

For $\varepsilon > 0$ we consider the minimization problem of finding $\sigma^{\eta}_{\varepsilon,h}$, belonging to a suitable finite element space V_h included in $H^m(\Omega_h)$, satisfying:

$$\forall v_h \in V_h, \quad \tilde{J}_{\varepsilon,h}(\sigma_{\varepsilon,h}) \le \tilde{J}_{\varepsilon,h}(v_h), \tag{17}$$

which is a discretization of (16). Let us mention that Apprato and Arcangéli (1991) showed that (17) is equivalent to the variational problem of finding $\sigma_{\varepsilon,h} \in V_h$ satisfying, for any $v_h \in V_h$:

$$\ell(\sigma_{\varepsilon,h} v_h) + \varepsilon(\sigma_{\varepsilon,h}, v_h)_{m,\Omega} = \ell(f v_h), \tag{18}$$

where $(u, v)_{m,\Omega} = \sum_{|\alpha|=m} \int_\Omega \partial^\alpha u \partial^\alpha v \, dx$. Apprato and Gout (1997) also showed that problems (17) and (18) have the same unique solution $\sigma_{\varepsilon,h}$, called the V_h-discrete smoothing D^m-spline of f relative to F and ε. Apprato and Arcangéli (1991) also validated the approach by assessing the accuracy of the approximant obtained for several analytical examples having a known reference solution.

Denoting by $M = M(h)$ the dimension of V_h and by $(\varphi_j)_{1\le j\le M}$ a basis of V_h, let us then define

$$\sigma_{\varepsilon,h} = \sum_{j=1}^{M} \alpha_j \varphi_j, \tag{19}$$

with $\alpha_j \in \mathbb{R}$, $1 \le j \le M$. Introducing the matrices $\mathcal{A} = (\ell(\varphi_i, \varphi_j))_{1\le i,j\le M}$, $\mathcal{R} = ((\varphi_i, \varphi_j)_{m,\Omega_h})_{1\le i,j\le M}$, and $\mathcal{F} = (\ell\,(f\varphi_i))_{1\le i\le M}$, we see that (18) is equivalent to the problem of finding $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_M) \in \mathbb{R}^M$ solution of

$$(\mathcal{A} + \varepsilon\mathcal{R}) = \mathcal{F}. \tag{20}$$

Regarding the numerical implementation of the algorithm, we choose to construct V_h following the ideas of Ciarlet (1978). Let $\tilde{V}_h$ be a finite element space constructed on $\tilde{\mathcal{Q}}_h$ such that $\tilde{V}_h$ is a finite-dimensional subspace of $H^m(\tilde{\Omega}) \cap C^k(\bar{\tilde{\Omega}})$, with $k = 1$ or 2. Let us also define V_h as the vector space of the restrictions to Ω_h of the functions of $\tilde{V}_h$. As an approximation of f, we can take the function $\Phi = \sigma_{\varepsilon,h|\,\Omega}$ that is in $H^m(\Omega) \cap C^k\left(\bar{\Omega}\right)$. We now have to determine in which sense Φ is an approximation of f. We use a result by Apprato and Arcangéli (1991) who proved that $\sigma_{\varepsilon,h}$ converges to f on F by establishing the theoretical error formula:

$$\left\|\sigma^{\eta}_{\varepsilon,h} - f\right\|^2_{0,F} \leq C\left(h^{2(m-1-\theta)} + \eta o(1) + \varepsilon\right), \tag{21}$$

when $\varepsilon \to 0$ and $\eta \to 0$, with $\theta \in\,]0, 1[$. Note that the parameter η comes from the quadrature formula used to approximate $\|\cdot\|_{0,F}$. This inequation gives a theoretical quantification of the error on the data set F. It is also possible to establish the convergence of the approximation on the entire domain Ω when the number of curves F_j tends to infinity (see Theorem 2.2 in Apprato and Gout, 1997 for a similar kind of data sets).

In most problems one would want to solve in practice, the value of m would be either 2 or 3, allowing one to get either a C^1 or a C^2 approximant. When $m = 2$, the finite elements used to solve the problem could typically be classical elements of class C^1 or C^2, such as the Argyris or the Bell triangle, or the Bogner–Fox–Schmit quadrangle (e.g., Ciarlet, 1978). When $m = 3$, one could use the same finite element of class C^2 as for $m = 2$. When $m > 3$, one could generalize the Bogner–Fox–Schmit quadrangle into a finite element of class C^{m-1}. Other elements, such as isoparametric finite elements or rational finite elements (e.g., Ciarlet, 1978) could also be used. Isoparametric finite elements are useful to impose boundary conditions, but this is not usually a critical problem in the context of surface approximation. On the other hand, the use of rational finite elements would lead to expensive calculations in terms of CPU time, therefore, we choose to use the Bogner–Fox–Schmit quadrangle of class C^1, which allows us to obtain a C^1-approximant. Note that in certain classes of interpolation problems, each data point must also be a node of the finite element grid, in which case the use of triangles, as opposed to quadrangles, greatly facilitates the creation of a suitable finite element mesh to numerically solve the problem. This is not the case in a surface approximation problem, in which we can select the finite element grid arbitrarily, which means that the use of quadrangles does not complicate the numerical algorithm in any way.

Let us also underline that a very significant advantage of the method introduced above is that we can arbitrarily select the degree of regularity of the final approximant. We could construct, if needed, a C^k-approximant with $k \geq 3$ (which could be useful in the context of image synthesis or ray tracing for example), by simply using a finite element space $V_h \subset H^m(\Omega) \cap C^k(\bar{\Omega})$.

APPLICATION TO SURFACE RECONSTRUCTION FROM BATHYMETRY SHIP TRACK DATA IN THE MARIANAS TRENCH

Detailed bathymetry maps are essential in several fields in geophysics, such as oceanography and marine geophysics. Historically, over the past decades, research vessels have collected a large number of depth echo soundings, also called SONAR (for “SOnic Navigation And Ranging”) bathymetry ship track data. Many of these measurements have been compiled to produce global bathymetry maps (e.g., Canadian Hydrographic Office, 1981). As underlined for instance by Sandwell and Smith (2001), in recent years tremendous advances in satellite altimetry have allowed researchers to produce very detailed bathymetry maps independently from satellite gravity field measurements. However, long-wavelength variations of the depth of the ocean floor are difficult to constrain using satellite altimetry, and ship track data are still often used instead for that purpose (Sandwell and Smith, 2001). It is, therefore, of interest to address the issue of producing a bathymetry map from a given set of SONAR bathymetry ship tracks. Let us mention that SONAR ship tracks are typically acquired as a discrete set of measurement points, as opposed to continuous recording. However, the typical horizontal interval between measurement points is always small compared to expected bathymetry variations; therefore, in the context of this study the data set can be considered as consisting of smooth continuous lines.

We select the region of the Marianas trench (Fig. 2). The trench is located in the North Pacific ocean, east of the South Honshu ridge, parallel to the Mariana

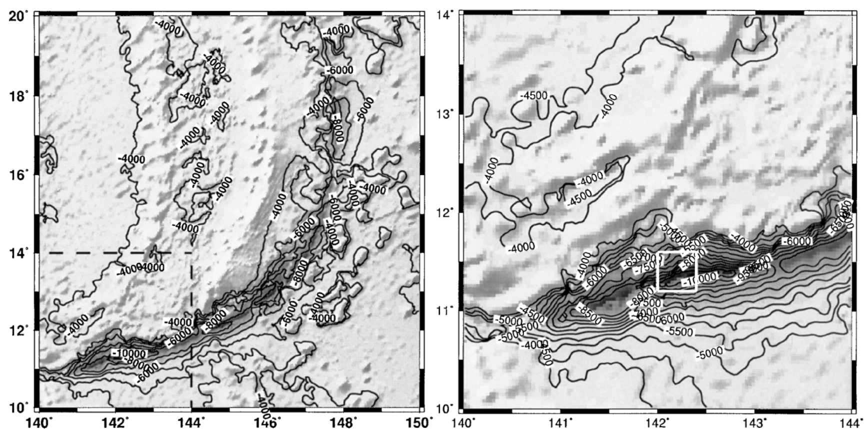

Figure 2. The Marianas trench (left) in the North Pacific ocean corresponds to the subduction zone at the contact between the Pacific and Philippine plates. It is the place on Earth where the oceans are the deepest, with a maximum slightly greater than 11 km in the region called “Challenger Deep” (left, dashed rectangle). The isolines represent depth in meters. On the close-up of this region (right), the white square represents the area where we test our surface approximation technique.

Islands. It corresponds to the subduction zone where the fast-moving Pacific plate converges against the slower moving Philippine plate. It is also the place on Earth where the oceans are the deepest, reaching a maximum depth of slightly more than 11 km in the so-called "Challenger Deep" area (Fig. 2, right). This region is ideal to test our surface approximation technique because it has been thoroughly studied; therefore, many ship track data sets are available. We select a 45 × 45 km area, corresponding to latitudes between 11.2° and 11.6° North, and longitudes between 142° and 142.4° East in Figure 2. We use 16 tracks from the database assembled by David T. Sandwell and coworkers at the University of California, San Diego (http://topex.ucsd.edu). Each individual track contains between 62 and 152 points giving depth for a given latitute and longitude. The total number of points in the whole data set is 1576. The depth varies between 6779 and 10952 m. As can be seen on Figure 3, the ship track coverage of the area is nonuniform. Note

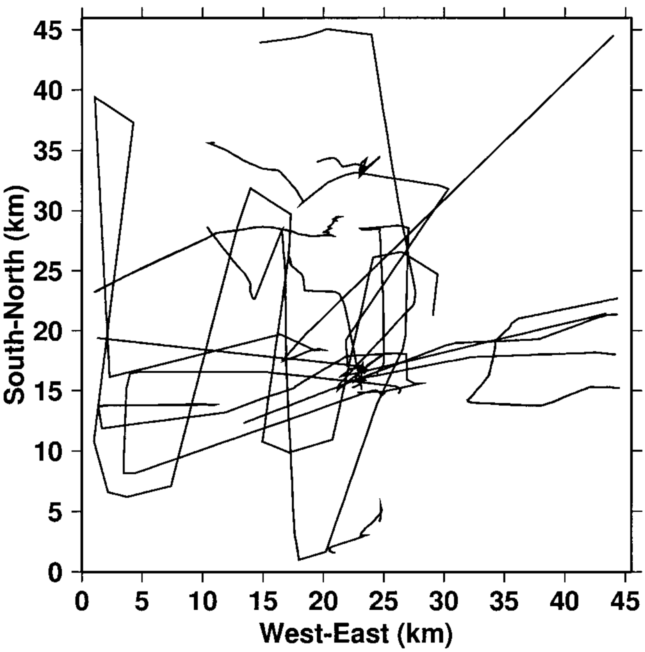

Figure 3. We focus on a 45 × 45 km region in the south-west of the Marianas trench of Figure 2. We use 16 bathymetry ship tracks, each containing between 62 and 152 points. The entire set of curves contains 1567 points. Each point gives depth for a given latitude and longitude. On this top view the coordinates have been mapped using the Universal Transverse Mercator (UTM) projection. The depth in the data set varies between 6779 and 10952 m. One can see that the ship track coverage is nonuniform. For instance we have little information in the north-east and south-east corners of the area.

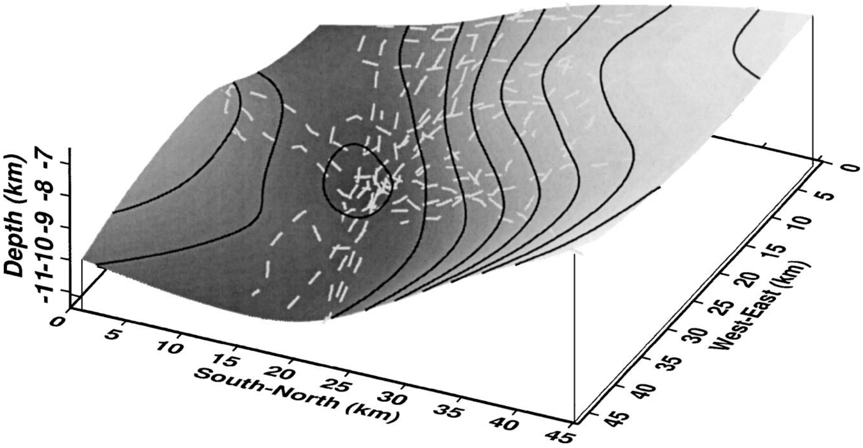

Figure 4. We construct a bathymetry map from the set of 16 ship track data curves of Figure 3 using a regular grid of 13 × 13 quadrangular Bogner–Fox–Schmit finite elements of class C^1. For display purposes, the approximant obtained has been evaluated on a regular 200 × 200 grid of points, and a vertical exaggeration factor of 3 has been applied. The original 16 ship tracks are also shown (dashed lines) to illustrate the quality of the surface obtained. The isolines represent bathymetry every 500 m from −10.5 to −7 km. By comparing with Figure 2, one can see that we are correctly reproducing the general trends of the bathymetry of the area.

in particular the lack of data in the north-east and south-east corners. Fortunately, data coverage is much better near the center in the deepest part of the trench.

We create an approximant using 169 quadrangular Bogner–Fox–Schmit finite elements defined on a regular 13 × 13 grid in the horizontal plane in the area under study. As underlined in the previous section, these elements allow us to obtain an approximant with C^1 regularity. Figure 4 shows a 3D view of the final surface obtained, as well as the original set of ship tracks. For display purposes, the approximant has been evaluated on a regular 200 × 200 grid of points and a vertical exaggeration factor of 3 has been applied. By comparing with Figure 2 and with the ship tracks, one can see that the smooth surface obtained correctly reproduces the general characteristics of the bathymetry of the region, and behaves satisfactorily even in the areas where data coverage is sparse.

We also evaluate the approximant obtained at the 1576 original data points of the 16 ship tracks (Fig. 5). The original ship track data and the approximated curves are almost superimposed, which illustrates that the technique is very accurate. To estimate the accuracy of the method more quantitatively, we evaluate the total quadratic error for the approximant based upon the classical formula:

$$\mathrm{Err}\,(\cup_i x_{3,i}) = \left(\frac{\sum_{i=1}^{1576} (\tilde{x}_{3,i} - x_{3,i})^2}{\sum_{i=1}^{1576} x_{3,i}^2} \right)^{1/2}, \tag{22}$$

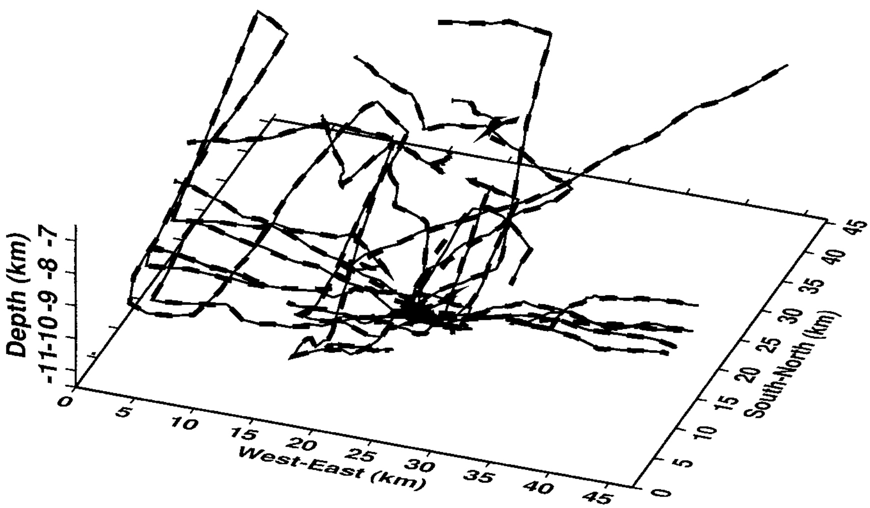

Figure 5. The original ship track data of Figure 3 (solid line) and the approximant of Figure 4 evaluated at the same 1576 points (thick dashed line) are almost superimposed. This illustrates the accuracy of our surface approximation technique. The total quadratic error is very small ($\varepsilon = 3.29 \times 10^{-5}$). For display purposes a vertical exaggeration factor of 3 has been applied on this 3D view.

where $x_{3,i}$ represents the x_3-data value, and where $\tilde{x}_{3,i}$ is the x_3-approximant value for the same $(x_{1,i}, x_{2,i}) \in \Omega$. We obtain a value of $\varepsilon = 3.29 \times 10^{-5}$, which is a very satisfactory result (unusually low in the context of surface approximation, e.g., Gout (1997); as a comparison, a usual D^m-spline (Arcangéli, 1986) applied to the same data set using the same finite-element grid gave an error of $\varepsilon = 6.4 \times 10^{-4}$, i.e., 20 times larger). The maximum error is of course located near the largest variations of the surface. In this regard, let us mention that in the case of a data set with large local variations, the method could be made even more precise, and the overall error reduced, by applying a preprocessing and postprocessing technique to the data, e.g., using scale transformations such as rank coding (Gout and Komatitsch, 2000) or splines under tension. An alternative approach, based on the use of additional first-derivative terms in the variational condition in order to minimize overshoots, was suggested by Hutchinson (1989).

CONCLUSIONS

We have introduced a new method to approximate a surface from a given set of curves, which allows one to take into account the continuous aspect of the data. The regularity of the surface obtained can be arbitrarily selected, i.e., it can be C^0, C^1, or higher. This allows us for instance to accurately describe the topography or bathymetry of real geophysical surfaces. We have used a ship track bathymetry data set from the Marianas trench to illustrate the method. Future work will focus on using quadrature formulas with a better order of approximation, and also applying the method to contour data and stream lines.

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