# A hybrid technique for 3-D modeling of high frequency teleseismic body waves in the Earth

Vadim Monteiller, Sébastien Chevrot, Dimitri Komatitsch and Nobuaki Fuji

IRAP - Observatoire Midi Pyrénées, CNRS UMR 5277 Toulouse, France
University of Toulouse and Institut universitaire de France, CNRS UMR 5563 GET, France

## Introduction

In the last decade, the deployment of dense regional arrays such as the USArray transportable array has considerably improved our capacity to image the interior of the Earth. However, the use of the wealth of information coming from these new and large amounts of high quality broadband data still heavily relies on asymptotic approaches. A finite-frequency approach has been used in a few tomographic studies at the regional scale, but for reasons of efficiency sensitivity kernels computed with the asymptotic approach (ray theory) introduced by Dahlen et al. (2000) were used, thus neglecting the near-field effects that are important in the vicinity of stations (Favier et al., 2004). In addition, this asymptotic approach to compute Fréchet kernels uses a spherically symmetric reference Earth model. In principle, these shortcomings can be overcome by full-wave approaches, which have been developed recently. However, these methods still suffer from a heavy computational cost, which limits their application to periods larger than a few seconds, even at the regional scale (Tape et al., 2009). Hereafter, we present a numerical method that allows us to model short period teleseismic waves in 3-D media, opening the possibility to perform waveform inversion of seismograms recorded by dense regional broadband arrays.

## Computation of traction in a Global Spherical model

Domain Reduction Methods are particularly attractive when the source is far from the local structures and if one wants to perform a sequence of simulations for this source with variable local structures. It is particularly relevant in high-resolution imaging based upon waveform inversion of teleseismic body waves at the regional scale. The Problem in then to compute wave propagation in a global spherically symmetric Earth model. For this purpose, normal mode summation has been widely used to compute long period synthetic seismograms because it provides accurate and complete solution of the wave equation. However, its main shortcoming is that the computation of spheroidal modes at periods below 8s is difficult, and that the number of modes that need to be summed increases dramatically with the frequency.
Thus, while normal mode methods are ideally suited to the computation of long period seismograms, they are not suitable to the modeling of high frequency teleseismic body wave records. In the 90s, new methods were developed to obtain exact solutions of the wave equation in a spherical Earth model. These methods solve systems of coupled first order ordinary differential equations (SODE, Eq 5) with respect to the radial coordinate for each expansion coefficient of the displacement vector expressed in the basis of the vector spherical harmonics. The GEMINI method (Green functions of the Earth by MINor Integration) directly solves for the expansion coefficients of displacements by numerically integrating spherical SODE (Eq 5) using second-order minors (Friederich & Dalkolmo, 1995).

In the frequency domain, the displacement field induced by a seismic wave is governed by the equation of motion:

$$-\rho\omega^2\mathbf{u} = \nabla . \sigma + \mathbf{f} \tag{1}$$

where $\rho$ is the density, **u** the displacement, $\sigma$ the stress tensor, and **f** the equivalent force due to a seismic source. To solve (1), it is useful to represent the displacement **u** in terms of scalar potentials:

$$\mathbf{u} = U\,\mathbf{r} + \nabla_1 V - \mathbf{r} \times \nabla_1 W \tag{2}$$

where $\nabla_1$ denotes the surface gradient:

$$\nabla_1=\theta\frac{\partial}{\partial\theta}+\varphi\frac{1}{\sin\theta}\frac{\partial}{\partial\varphi} \tag{3}$$

Each potential in (2) can be expanded in fully normalized spherical harmonics. For example:

$$\mathrm{U}(\mathrm{r},\theta,\varphi;\omega)=\sum_{l=0}^{l=\infty}\sum_{m=-l}^{m=l}\mathrm{U}_l^m(\mathrm{r},\omega)\,\mathrm{Y}_l^m(\theta,\varphi) \tag{4}$$

Where $\mathrm{Y}_l^m$ is the spherical harmonic function of degree $l$ and azimuthal order $m$. From the equation set (1) - (4), we obtain a set of coupled ordinary differential equations for displacement and stress:

$$\frac{\mathrm{d}}{\mathrm{d\,r}}\mathbf{y}(r)=\mathbf{A}(r)\mathbf{y}(r)+\mathbf{s}(r) \tag{5}$$

where **y** is a vector containing displacement and stress potentials $(\mathrm{U}_l^m,\mathrm{R}_l^m,\mathrm{V}_l^m,\mathrm{S}_l^m)$ for spheroidal motion, or $(\mathrm{W}_l^m,T_l^m)$ for toroidal motion. The excitation vector **s** contains the expansion coefficients of the source force potentials. We have modified the GEMINI software in order to compute and store the solution of (5) at depth, because in its initial form the software only computed the displacements at the free surface. Once the displacement vector is obtained, we use it to compute the traction vector on the sides of a local grid. First, we obtained the strain tensor $\varepsilon$ from the displacement by computing the partial derivatives of spherical harmonics with respect to the colatitude $\theta$ and longitude $\varphi$. We then obtain the components of the stress tensor by:

$$\sigma=\mathrm{C}:\varepsilon \tag{6}$$

from which we can compute the traction $\mathbf{t} = \sigma \cdot \mathbf{n}$ on any surface defined by its unit normal **n**.

## Coupling 1-D global and 3-D regional numerical modeling technique

The 3-D regional model that we want to study in detail is embedded in a 1-D spherically-symmetric global model. We build a mesh that includes the whole 3-D model and extends into part of the 1-D model, and thus the material properties on its edges are those of the 1-D model. To simulate seismic wave propagation in that mesh, we need to introduce the incident field corresponding to teleseismic sources (epicentral distance larger than 3000 km) and also need to absorb the outgoing waves diffracted by the 3-D model. To do that, we follow the method suggested by Bielak & Christiano (1984), in which absorbing boundary conditions are applied to the diffracted field only. We write the total displacement vector $\mathbf{u}$ as the sum of the incident wave field $\mathbf{u_0}$, which is known numerically at all the mesh points and at each time step from the GEMINI calculations performed in the global 1-D model, and the diffracted field $\mathbf{u}_d$. We compute the total field in the mesh using the SEM and then apply the absorbing boundary condition to $\mathbf{u}_d=\mathbf{u}-\mathbf{u_0}$. We currently use the paraxial absorbing boundary condition of Stacey (1988). This approximate boundary condition relates traction to velocity:

$$\mathbf{t}=\rho[v_n(\mathbf{n}.\partial_t\mathbf{u}_d).\mathbf{n}+v_1(\mathbf{t_1}.\partial_t\mathbf{u}_d).\mathbf{t_1}+v_2(\mathbf{t_2}.\partial_t\mathbf{u}_d).\mathbf{t_2}] \tag{7}$$

where $\mathbf{t}_1$ and $\mathbf{t}_2$ are orthogonal unit vectors tangential to the absorbing boundary with outward normal $\mathbf{n}$, $v_n$ is the quasi-P wave speed of waves traveling in the direction $\mathbf{n}$, $v_1$ is the quasi-S wave speed of wave polarized in the $\mathbf{t}_1$ direction, and is the quasi-S wave speed of waves polarized in the $\mathbf{t}_2$ direction. Using PML Perfectly Matched absorbing Layers would be more efficient and will be implemented in future work. Implementing the paraxial absorbing condition to the diffracted field only implies knowledge of the traction vector and of the displacement vector corresponding to the incident wave field on all the absorbing edges of the grid because they must be subtracted from the total field before applying the absorbing condition; we compute them using a modified version of the GEMINI code described above because the material properties along those edges are those of the 1-D model. In the 3-D regional domain, we use a spectral element method (see e.g. Cohen (2002), Tromp et al. (2008)), which is a highly accurate technique to model seismic wave propagation in elastic or anelastic media. The SEM is based upon the variational (or weak) form of the seismic wave equation. Because it uses high-degree polynomial basis functions, it can handle distorted meshes (Oliviera & Seriani, 2011) and does not necessitate interpolation of material properties, it is highly accurate and allows us to include all the complexity that may affect the seismic wave fields: the topography of the free surface and of internal discontinuities, anelasticity, anisotropy, and lateral variations of elastic parameters and density.

## Validation

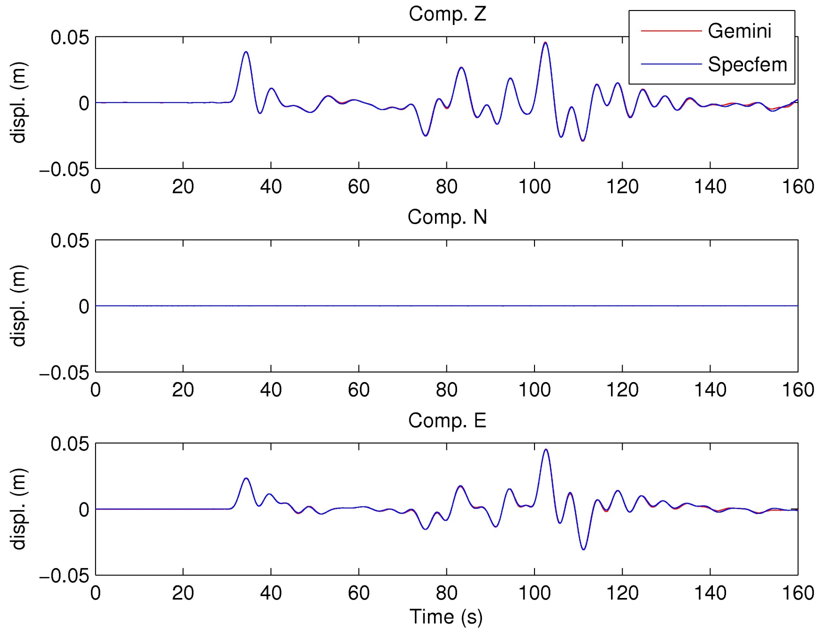


***Figure 1*** *Comparison of displacement seismograms (up to 5s) computed at the free surface with the GEMINI method and spectral element method (SPECFEM3D). The source is an explosion located at 30° epicentral distance and 200 km depth. The embedded model inside the local box is the same as that used for the teleseismic propagation.*

In order to test the code we generated waveforms for a wide range of trials models. We show here an example using the spherical model inside a 2-degree wide regional box. We computed the displacement at the free surface with the GEMINI method and spectral elements method. As shown in Figure 1, the results are very close, which validates the method.

## Conclusions

We developed an hybrid method to model short period teleseismic waves in 3-D media. We solve the equations of motion directly in the frequency domain to propagate teleseismic waves from the source to a 3D regional domain. The knowledge of traction and displacement fields along the edges of the domain allows us to use a spectral element method inside this domain and thus model the full 3D wave field. Each of these two approaches has the advantage of requiring modest computational resources, opening the possibility to perform waveform inversion of seismograms recorded by dense regional broadband arrays.

## References


Bielak, J. & Christiano, P., 1984. On the effective seismic input for non-linear soil-structure interaction systems, *Earthquake Eng. Struct. Dyn.*, **12**, 107-119.

Cohen, G., 2002. *Higher-order numerical methods for transient wave equations*, Springer-Verlag; Berlin, Germany.

Dahlen, F.A., Hung, S.H. & Nolet, G., 2000, Fréchet kernels for finite-frequency traveltimes-I. Theory. *Geophys. J. Int.* **153**, 213-228

Favier, N., Chevrot, S. & Komatitsch, D., 2004, Near-field influences on shear wave splitting and traveltime sensitivity kernels, *Geophys. J. Int.*, **156**, 467-482.

Friederich, W. & Dalkolmo, J., 1995, Complete synthetic seismograms for a spherically symmetric earth by numerical computation of the green's function in the frequency domain, *Geophys. J. Int.*, **122,** 537-550.

Komatitsch, D. & Tromp, J., 1999. Introduction to the spectral-element method for 3-D seismic wave propagation, *Geophys. J. Int.*, **139**(3), 806-822.

Oliveira, S. P. & Seriani, G., 2011. Effect of element distortion on the numerical dispersion of spectral element methods, *Communications in Computational Physics*, **9**(4), 937-958.

Stacey, R., 1988. Improved transparent boundary formulations for the elastic wave equation, *Bull. Seismol. Soc. Am.*, **78**(6), 2089-2097.

Tape, C., Liu QY., Maggi A. & Tromp J., 2009, Adjoint tomography of the Southern California crust, *science*., **325**, 998-992.

Tromp, J., Komatitsch, D., & Liu, Q., 2008. Spectral-element and adjoint methods in seismology, *Communications in Computational Physics*, **3**(1),1-32.